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Working Paper 1439-4

Investigation of system input and output blending for zero placement

Started: November 25, 2014
Latest Revision: November 25, 2014

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Prepared for

NASA NRA Grant:
Performance Adaptive Aeroelastic Wing
Contract No. NNX14AL36A

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1.0 INTRODUCTION

The zeros are a function of the system input and output. Appropriate selection of sensor outputs and control inputs influences the zeros of the open loop system and this directly influences the effectiveness of any resulting controller. The purpose of this working paper is to summarize the investigation of appropriate input and output blending for zero placement as applied to an aeroseroelastic pitch-plunge model. The material in this working paper was taken directly from Ref. 1.

2.0 PITCH-PLUNGE MODEL

2.1 Model Description

For this investigation, the simple pitch-plunge model was used (Figure 1). This model is equipped with two inputs and three outputs.

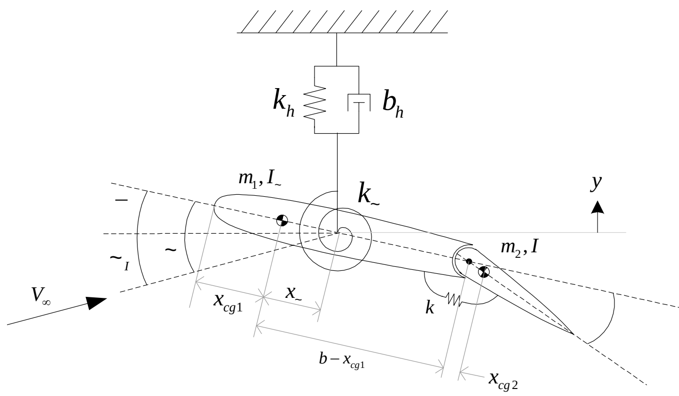

Figure 1: Pitch-plunge model.

The inputs to the system are the inertial airflow angle of attack (~$_I$) and the input control surface deflection ($_0$). The outputs are the system states: the plunge deflection (y), the pitch deflection (¯) and the control surface deflection from nominal (¢).* The system used for this study has a constant freestream velocity of 15 m/s.

2.2 Nominal System Dynamics

The complete system has 2 inputs and 3 outputs; therefore the complete system is MIMO and can be represented as a 3 × 2 transfer function matrix with 6 individual SISO LTI systems. Each of the 6 system root loci is displayed in Figure 2 through Figure 7.

* The total control surface defection is = $_0$ + ¢

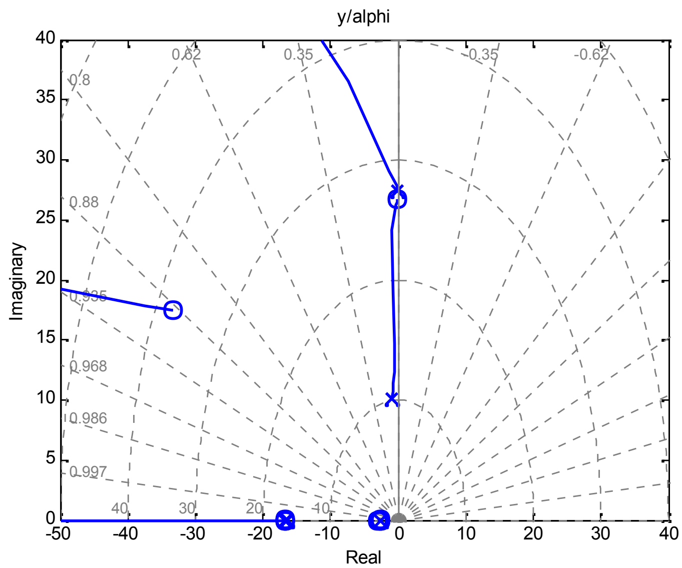

Figure 2: *y*/~$_I$ root locus.

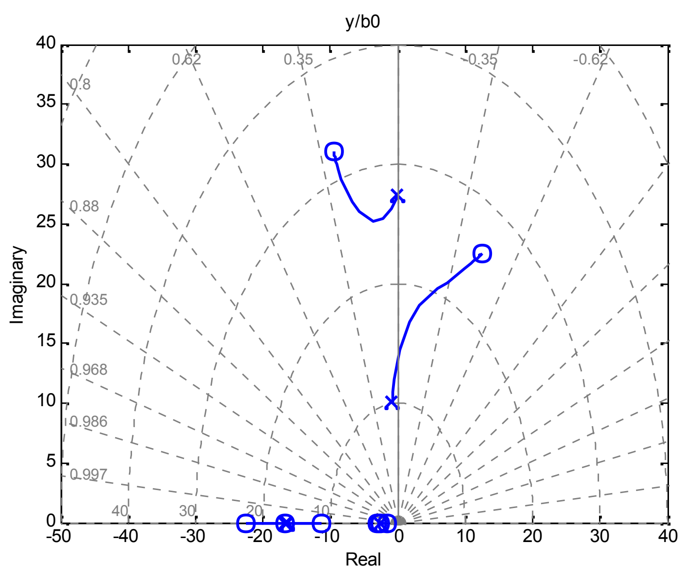

Figure 3: *y*/ $_0$ root locus.

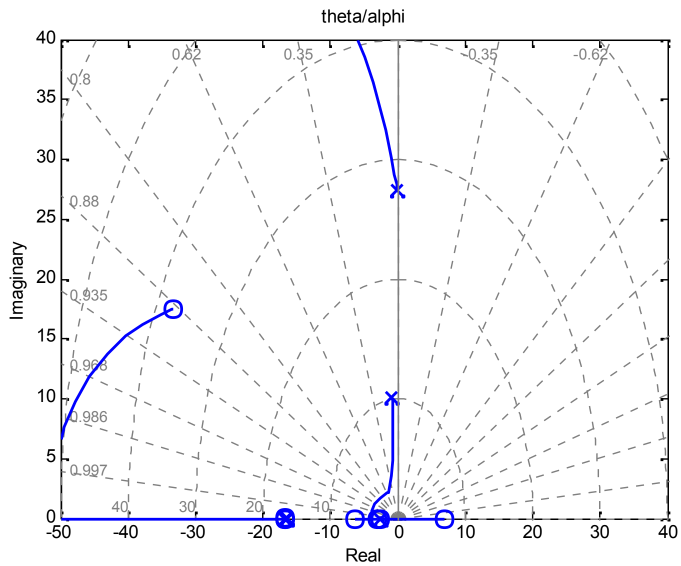

Figure 4: ¯/~$_I$ root locus.

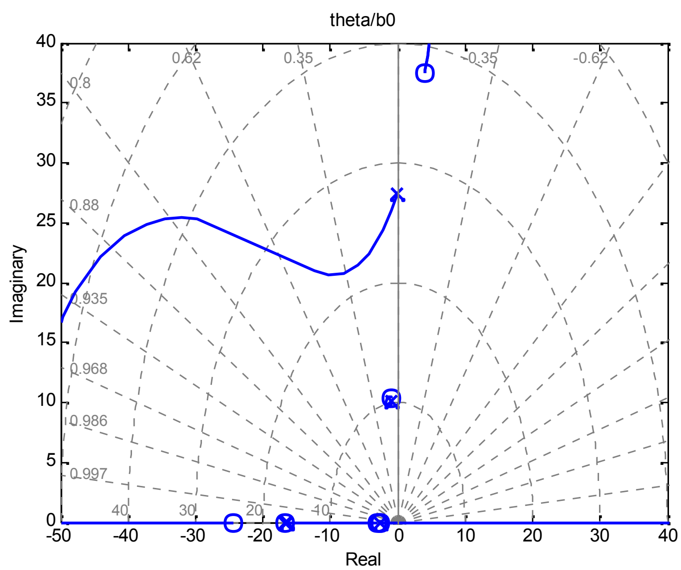

Figure 5: ¯/ $_0$ root locus.

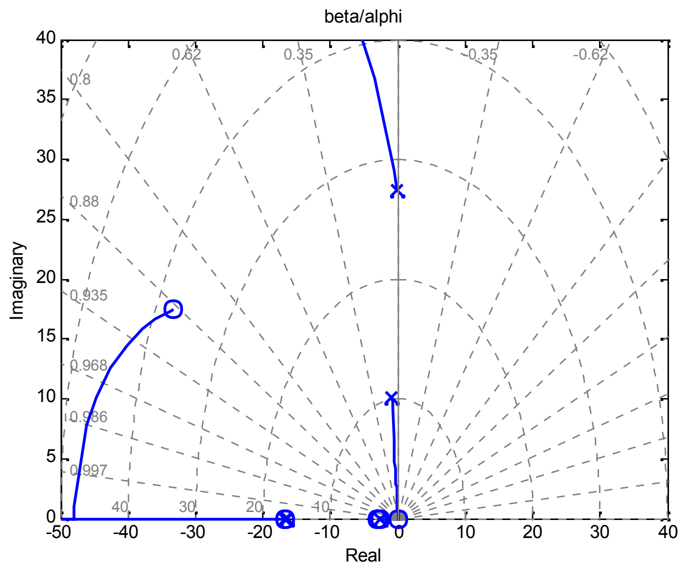

Figure 6: ¢/~$_I$ root locus.

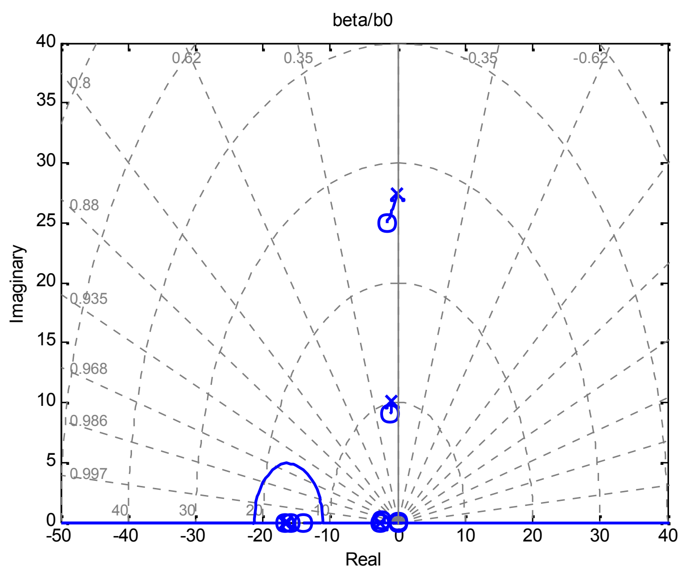

Figure 7: ¢ / $_0$ root locus.

It is evident that the closed loop behavior varies significantly between each of the above systems due to the varying zero locations.

3.0 INVESTIGATION OF SENSOR OUTPUT BLENDING

A SISO system can be constructed that is a linear combination of the above systems. The resulting linear combination will have different zeros than either of the two individual systems. Assume a SISO system that is a linear combination of two systems with the same input. This is represented in Eq. (1) with the $y/\ _0$ and $^{-}/\ _0$ transfer functions.

$$SYS = l_1 \frac{y\diagup_0}{\Delta} + l_2 \frac{\overline{\ }\diagup_0}{\Delta} \qquad (1)$$

A zero locus, which is a function of the coefficients (l_1 and l_2) can be constructed.[2] To perform this, Eq. (1) is rearranged (Eq. (2)).

$$
\begin{aligned}
SYS &= l_1 \frac{y/_0}{\Delta}\left(1+\frac{l_2}{l_1}\frac{-/_0}{y/_0}\right) \\
&= l_1 \frac{y/_0}{\Delta}\left(1+kSYS_z\right) \qquad (2) \\
where: k &= \frac{l_2}{l_1} \quad and \quad SYS_z = \frac{-/_0}{y/_0}
\end{aligned}
$$

The root locus of SYS_Z will be the locus of zeros for a system that is a linear combination of the two system outputs. For the two systems shown, this zero locus is displayed on Figure 8 below. The solid locus is the zero location as k varies from zero to infinity. The dotted locus is the zero location as k varies from zero to negative infinity.

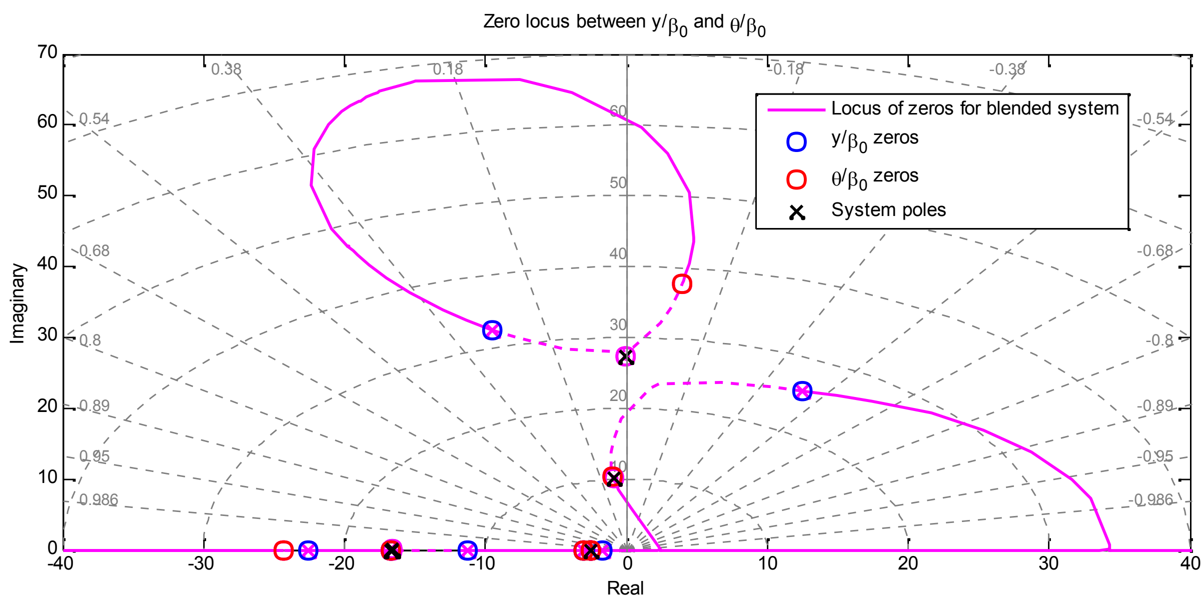

Figure 8: Sample zero locus between two systems with the same input.

This blending concept can be extended to three or more systems. Assume that a blended system is a linear combination of n systems (Eq. (3)).

$$
\begin{aligned}
SYS &= l_1 \frac{SYS_1}{\Delta} + l_2 \frac{SYS_1}{\Delta} + \ldots + l_r \frac{SYS_r}{\Delta} \\
&= \begin{bmatrix} l_1 & l_2 & \cdots & l_r \end{bmatrix} \begin{bmatrix} \frac{SYS_1}{\Delta} \\ \frac{SYS_1}{\Delta} \\ \vdots \\ \frac{SYS_r}{\Delta} \end{bmatrix}
\end{aligned}
\tag{3}
$$

Rearranging similar to Eq. (2) yields:

$$
\begin{aligned}
SYS &= l_1 \frac{SYS_1}{\Delta} \left(1 + \frac{l_2}{l_1} \frac{SYS_2 + \frac{l_3}{l_2} SYS_3 + \ldots + \frac{l_r}{l_2} SYS_r}{SYS_1} \right) \\
&= l_1 \frac{SYS_1}{\Delta} \left(1 + k_1 SYS_z \right) \\
where: k_1 &= \frac{l_2}{l_1} \quad and \quad SYS_Z = \frac{SYS_2 + k_2 SYS_3 + \ldots + k_{r-1} SYS_r}{SYS_1} \\
and: k_i &= \frac{l_{i+1}}{l_2} \quad \forall i = 2, 3, \ldots, r-1
\end{aligned}
\tag{4}
$$

The zero loci for a system that is the linear combination of 3 or more systems is slightly more complicated. Figure 9 below displays an example which is a linear combination of $y/_{0}$, $¯/_{0}$ and $¢/_{0}$. The result is a series of loci. Note that the locus is Figure 8 corresponds to the $k_2 = 0$ locus in Figure 9 below. When $k_2 = 0$, the linear combination involves only two systems: $y/_{0}$ and $¯/_{0}$.

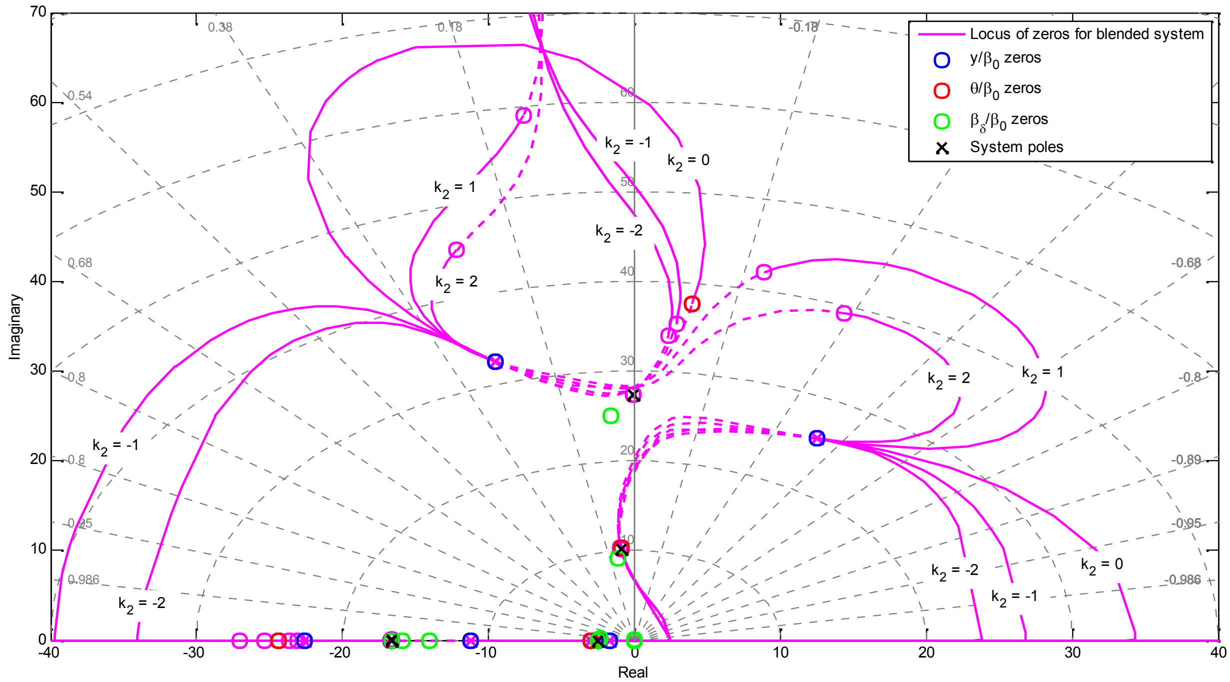

Figure 9: Zero locus of a system that is a linear combination of *y*/ 0, ¯/ 0 and ¢/ 0.

Much flexibility is added when it is assumed that there are 3 or more outputs to blend. The zeros can take on values of a much more broad range. There are still constraints on where the zeros can be since all zeros are related by the linear combination. As the gains are varied to ideally place one zero, all other zeros are affected. A locus showing all possible zero locations with the three outputs is displayed in Figure 10 below.

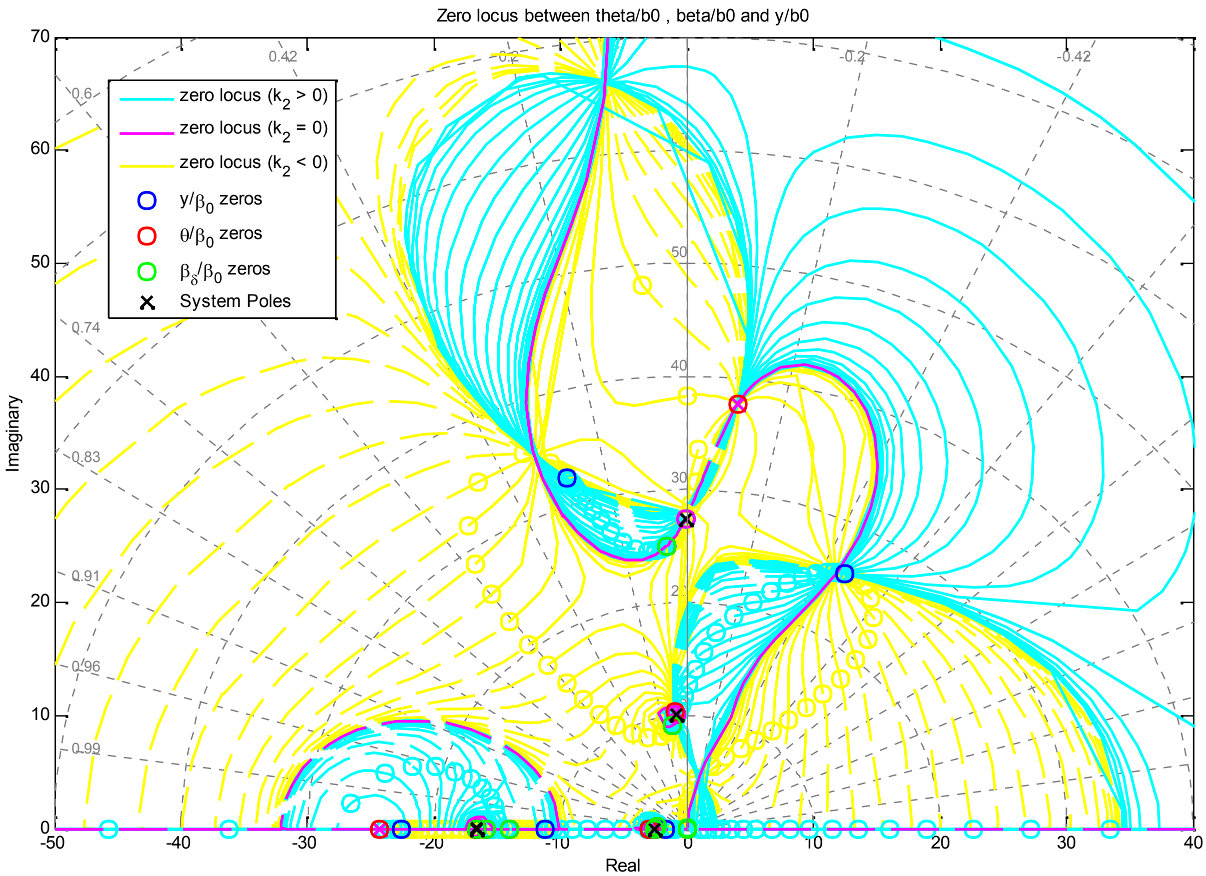

Figure 10: Zero locus of a blend between three system outputs.

4.0 INVESTIGATION OF CONTROL INPUT BLENDING

A similar investigation can be performed when it assumed that multiple control inputs are available for blending. Conceptually, the process is identical but the linear combination is with systems that have the same output but different inputs. A resultant system that is the linear combination of two systems with the same output but different inputs is displayed in Eq. (5) below.

$$SYS = m_1 \frac{{}^{y}\!/_{0}}{\Delta} + m_2 \frac{{}^{y}\!/_{\sim I}}{\Delta} \tag{5}$$

Extending the concept to 3 or more systems results in Eq. (6).

$$\begin{aligned} SYS &= m_1 \frac{SYS_1}{\Delta} + m_2 \frac{SYS_1}{\Delta} + \ldots + m_p \frac{SYS_r}{\Delta} \\ &= \begin{bmatrix} \frac{SYS_1}{\Delta} & \frac{SYS_1}{\Delta} & \cdots & \frac{SYS_r}{\Delta} \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_p \end{bmatrix} \end{aligned} \tag{6}$$

As was done for the sensor output linear combination, this can be rearranged.

$$SYS = m_1 \frac{SYS_1}{\Delta}\left(1 + \frac{m_2}{m_1}\frac{SYS_2 + \frac{m_3}{m_2}SYS_3 + \ldots + \frac{m_p}{m_2}SYS_p}{SYS_1}\right)$$

$$= m_1 \frac{SYS_1}{\Delta}\left(1 + k_1 SYS_z\right) \qquad (7)$$

$$where: k_1 = \frac{m_2}{m_1} \quad and \quad SYS_Z = \frac{SYS_2 + k_2 SYS_3 + \ldots + k_{p-1}SYS_p}{SYS_1}$$

$$and: k_i = \frac{m_{i+1}}{m_2} \quad \forall i = 2,3,\ldots,p-1$$

Examples will not be displayed since the zero locus of a linear combination of systems was already adequately shown in the preceding section.

REFERENCES

1 Danowsky, B., and Thompson, P., "Investigation of System Input and Output Blending For Aeroservoelastic Suppression," *Systems Technology, Inc.*, STI WP 1398-3, May 2010.

2 Chan, S. Y., P. Y. Cheng, D. M. Pitt, T. Myers, D. H. Klyde, R. E. Magdaleno and D. T. McRuer, "Aeroservoelastic Stabilization Techniques for Hypersonic Flight Vehicles," NASA CR-187614, Sept. 1991.